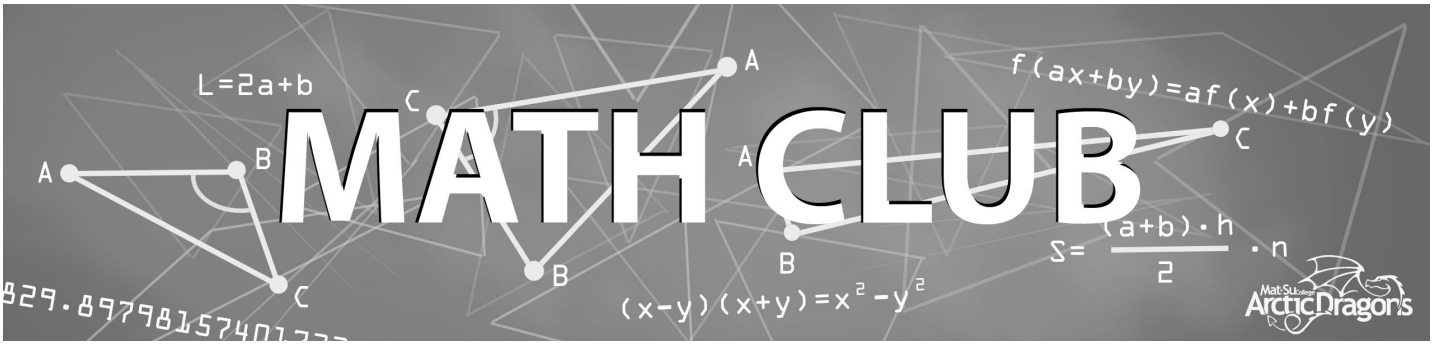

Problem Set of the Week

Problem 1 - Arithmetic **(⋆)**

For how many prime numbers p is the number $p^4 + 1$ again a prime number?

Problem 2 - Arithmetic **(⋆ ⋆)**

The number $3^{32} - 1$ has exactly two divisors between 75 and 84. What are they?

Problem 3 - Algebra **(⋆ ⋆ ⋆)**

At the University of Perfect Reasoning, each college has a dean and each dean has an apprentice. At least one apprentice is a thief. To remedy this situation without embarrassment, the chancellor of the university proclaims the following true statements:

1. At least one apprentice is a thief.
2. Every thief is known to be a thief to every one except to his or her own employer, that is dean, and all deans reason perfectly.
3. If n days from today you have concluded that your apprentice is a thief, you will publicly denounce your apprentice on that day.

If in fact $k \geq 1$ of the apprentices are thieves, when will they be denounced, and how do the deans reason?

Rules: Solve one problem or solve them all. Submit solutions to Dr. Luke Grabarek in Snodgrass Hall 103A or via e-mail at lgrabarek@matsu.alaska.edu. All submissions will be awarded a $\star$ and, in addition, correct solutions receive the $\star$ rating of the problem.

"I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics, for men thus endowed seem to have an extra sense." - Charles Darwin